MAINTAINING THE SERVICE LEVEL BY USING THE COORDINATED ORDER MODEL FOR THE REPLENISHMENT OF DISTRIBUTION STOCKS

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Abstract
The distribution companies coordinate the multi-item replenishment orders in the view of truck transport to benefit from major cost savings. Yet, the moment of order launching and item allocation can generate stock surplus or shortage, preventing these companies from reaching the established service level. The current study presents a mathematical model introduced by Carlson and Miltenburg and applied at first to the wooden products transport. The authors extend the field of application to the case of a tire distribution company that orders multiple similar items, the amount of the total order representing a full truckload, from a single distribution point. The results of the model allows establishing the moment of order launching and the volume of the order for each item on the basis of balancing the real service level with the service objective, offering thus to the company a favourable competitive position.

Keywords: replenishment policies, service level, tires, multi-item inventory models, coordinated order

JEL Classification: M11, C61

Introduction
The authors’ interest in the field of finite products distribution is not at all casual. Their meetings during training and consultancy activities with specialists in this area raised the awareness of the importance of their work for the end consumer and the needs to obtain more efficiency so that this activity should not generate unjustified costs. In the context of a fiercer competition, the distributors should satisfy the clients, who analyze at the same time the quality of the products, the required price and, not in the least, the promptness of the delivery. Although the coordinated order of multiple products is profitable from the point of view of transportation costs, it can lead to stock surplus of certain items, or to stockouts for

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the items with low demand. Besides, if we also add the random character of the demand, the managerial decision of order launching (“how much” and “when”) is difficult to make.

The service point model for stock replenishment, taking into account the coordinated order of multiple similar products transported by truck and using the full transportation capacity, from a single distribution point, helps to make this decision. The model was proposed at first by Carlson and Miltenburg (1988), who successfully applied it for the coordinated order of some wooden products. The current paper clarifies the elements that the distributor should know for supporting the replenishment decision with the model: the length of review period, the time of launching the order, the allocation of the truckload among the individual items of the family, the service level.

There are also specified the hypotheses of the model, defined the level of acceptance of the stock shortage and the real average aggregated level of the stock shortage, as well as an analytical form to express the latter in the case of a normal distribution of demand.

At the theoretical level, an algorithmic procedure was defined by the initiators of the model both for establishing the optimal moment for launching the order, and for identifying the order quantity on types of products, which is recommended by the authors to the distribution companies.

The practical illustration of the described model is based on the information offered by a company that distributes a portfolio of tires acquired from a single distribution point, using full truckloads.

Processing the information led to obtaining elements for supporting the replenishment decision: identifying the optimal moment for launching the order, pointing out the quantity that should be ordered for each type of tires so that the established service level should be met, under the circumstances of enhanced efficiency created by the coordinated order of a high volume of products: reducing the transportation costs or taking advantage of quantity discounts.

1. The Service Point Model

1.1 The Multi-Item Replenishment System with a Truckload

Although the analysis and the decisions of stock replenishment are usually made for individual items (Firiciă, 2002), in practice the distributor orders a variety of similar items from the same point that should be shipped by the same transport. Thus, shipping various car tires from a far away single supplier is much more economical if it is coordinated by launching the order of a big quantity of items of a family at the truckload level. This way, the aggregate quantity ordered is imposed and predefined. Then it should be decided the order size for each item. The coordinated ordering system has several advantages:

• The aggregate ordering-storage cost decreases, including the transport from the supplier. In the case of tires, the ordering cost is insignificant, while the major component of the aggregate cost is the shipping cost. It is important to benefit of economies of scale when ordering a full truckload of tires and to allocate the capacity in a way that could reduce costs.
• The distributor can benefit from price discounts according to the quantity.
• A certain rhythmical character of the orders can be obtained, leveling thus the random character of the demand or the different lead time for individual products of the same family. The lead time is the time elapsed from the moment at which it is decided to place an order, until it is physically on the shelf ready to satisfy customer demands. The existence of such an interval requires the insurance of sufficient inventory to satisfy demand during this period.
• When ordering overseas, it is often necessary to fill the shipping container to keep shipping costs under control.

Usually the order for the whole product family is caused by the inventory decrease of a single item under a certain level. Despite its apparent simplicity this system does not correspond to any of the ordinary control systems: with continuous or periodic review. Yet, the periodic review, at the same \( R \) period for all the products of the family is advisable in this case, taking into account the advantage of the both contracting parties: supplier and distributor. However, the random character of the demand makes it difficult establishing the review period, the appropriate moment of launching the order, and the replenishment size allocation. In a survey, Aksoy and Erenguc (1987) have shown that the existing models are complex and require computational effort to obtain the exact solution. Literature relies mainly on heuristic procedures. Carlson and Miltenburg (1988) introduced and used the service point model in industrial applications, and gave solutions for the appropriate time of launching the order in a periodic review system. Miltenburg (1985) has shown how to allocate the order to different items. In their comprehensive book, Silver, Pyke, and Peterson (1998) devoted a whole chapter to the issue of coordinated replenishments from a single point. The current study implements both the service level model and truckload capacity allocation in tire distribution of a Romanian private company. The objective is to illustrate the way a company can satisfy the demand, maintaining at the same time an appropriate service level. This way, the company improves the relationship with customers and gets competitive advantage. At the same time, the transport coordination of multiple types of tires leads to cost savings by truck transport.

1.2 Main Decisions in Coordinated Control Context

In order to avoid both the inventory shortage and surplus of certain individual items of the same family, the distributor should have clear criteria for launching an order. Silver, Pyke, and Peterson (1998) show that, at a first analysis, the distributor will make decisions regarding:

- \textit{The length of the R review period of the inventory.} The random character of the demand for each item of the family makes difficult the decisions concerning the establishing of the \( R \) review period, because the launching of an order for a certain product triggers the order of the other products of the family.

- \textit{The appropriate time of launching the order.} If the review reveals that it is not necessary to replenish the inventory corresponding to the product family, one should wait until the next review. Consequently, the safety stock for each item should satisfy the demand for a longer period than \( R \) review period to hedge against stockout during both the time between reviews and the lead time.
Allocating the truckload among the individual items of the family. The decision upon the general amount of order is imposed by the truck transport capacity. It still has to be made the allocation for each individual item of the quantity ordered at the review moment. The truckload must be allocated among the individual items in the family.

The level of service. The distributor decides upon the appropriate level of satisfying his clients, taking into account the demand fluctuation.

The way to measure service is to define the fraction of customer demand filled off-the-shelf without backorders or lost sales. This fraction can actually be viewed as the probability that an incoming order is met routinely from the shelf, and is also known as the fill rate. The level of fill rate is a matter of managerial decision and may be set based on those fill rates provided by the competition or the industry level. The timing of launching an order is based on the decision to balance the actual expected service level against the service objective.

1.3 The Service Point Model – How to Make the Replenishment Decision

The service point model (Carlson, Miltenburg, 1988) is applicable in this situation. It is supposed that both the decision upon the review period and service level have already been made. For a single item, the \((s, Q)\) reorder model has been widely applied to determine both the time of reorder and the replenishment quantity (Ritzman, Krajewski, 2003). The service point model develops a replenishment policy for multiple items with probabilistic demand when replenishment is coordinated for a family of items from the same supplier and comes with trucks, the main component of inventory cost being transportation cost from the supplier. Procedures are developed to determine the “when to order” rule. A separate procedure similar to “how much to order” is suggested for the allocation of the truckload among the individual items in the family. The assumptions of the model are the following:

- The replenishment of the \(n\) items is coordinated from a single stocking point, by using a full truckload. The family replenishment size is the truck capacity, \(V\).
- The periodic review system is in place, at every \(R\) periods. The value of \(R\) is dictated by external factors – as the frequency of truck deliveries.
- The lead time \(L\) for all items in the family is constant.
- The demand for all the items during the \(R+L\) period is random, follows a normal Gaussian distribution and the mean and standard deviation are known or can be estimated from sample data.
- Demand distributions for the various items are independent of each other.

Notations:

\(R\) = the review interval pre-specified (expressed in time periods)

\(L\) = the lead time (expressed in the same units as \(R\))

\(p\) = the fraction of demand that is met by on-hand inventory, without backorders, or the fill rate
The distributor must decide on the acceptable aggregate level of shortages per replenishment cycle, $AL$. Carlson and Miltenburg (1988) propose the following acceptable level:

$$AL = \sum_{i=1}^{n} Q_i \left(1 - P_i\right)$$  \hspace{1cm} (1)$$

This aggregate level states the upper bound of shortage calculated per family of items the company could afford in order to meet customer needs without backorders. In this coordinated multi-item model, the acceptable level $AL$ has a triggering function. At the review time, the distributor must analyze the available stock characteristics for every item in the family in order to determine the expected shortage in a replenishment cycle per family. If the decision for the current review time is do not order, then the distributor should have enough stock to meet the expected demand during the next $R+L$ periods. An aggregate measure for expected shortage during the next $R+L$ periods $ES(R+L)$ is defined by

$$ES(R+L) = \sum_{i=1}^{n} \int_{y_i}^{\infty} \left(x - y_i\right)f_i(x)dx$$  \hspace{1cm} (2)$$

where $y_i$ is the inventory position of item $i$ at the time of the review, and $f_i(x)$ is the probability density function of the normal distribution, modeling the demand for item $i$ through the time span $R+L$ (assumption 4).

The decision rule is to reorder a full truck load if $ES(R+L) > AL$, or otherwise delay ordering.

### 1.4 Derivation of the Expected Shortage Function with Normal Demand

The model for one item with normal demand during the lead time $L$ provides the reorder level

$$s = \bar{x} + k \cdot \sigma$$  \hspace{1cm} (3)$$

where $s$ is the reorder point, $\bar{x}$ and $\sigma$ are the expected demand and standard deviation of demand during the lead time, or their estimated values. $k$ is a safety factor, closely related to the pre-determined service level $\alpha$, solution to the equation

$$F(k) = \alpha$$


with
\[ F(k) = \int_{-\infty}^{k} f(x) \, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{k} e^{-\frac{x^2}{2}} \, dx \]

the cumulative density function of the standardized normal distribution. The safety factor \( k \) is
\[ k = \frac{s - R}{\sigma} \]

The available stock is compared with the reorder point \( s \). When the available stock falls below the reorder point, replenishment is triggered. The expected shortage \( ES \) in this one-item situation is
\[ ES = \frac{1}{\sigma L \sqrt{2\pi}} \int_{-\infty}^{\infty} (x - s)e^{-\frac{(x-s)^2}{2\sigma^2}} \, dx \]
(4)

Defining for any real \( k \) the function
\[ G(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{k} (x - k)e^{-\frac{x^2}{2}} \, dx \]
(5)

and making usual transformations, \( ES \) becomes
\[ ES = \sigma L G(k) \]
(6)

As a conclusion, the expected shortage depends on the service level by means of the \( k \) factor and the standard deviation of demand during the lead time, or its estimation. Another format of \( G(k) \) is
\[ G(k) = f(k) - k[1 - F(k)] \]
where
\[ f(k) = \frac{1}{\sqrt{2\pi}} s^{-\frac{k^2}{2}} \]

\( G(k) \) proves to be a strictly decreasing function, having the property
\[ G(k) - G(-k) = -k \]
(7)

In the multi-item periodic review context, for each individual item two points of variation should be noted:
- If the order is not launched at the time of review, the inventory should satisfy the customer needs of that item over the next \( R+L \) periods, not only during \( L \) periods.
- The inventory position of the individual item \( i \) at the time of review is not necessarily the reorder point. However, the “safety factor” \( k \) can still be calculated:
\[ k_i = \frac{2L - \bar{R}_i + L_i}{\sigma_{R+L_i}} \]
(8)
for \( i = 1, 2, \ldots, n \), where \( y_i \) is the inventory position at the time of review, \( R+L \) is the forecast or expected demand over the review interval plus a replenishment lead time, and \( \sigma_{R+L} \) is the standard deviation of demand over the same interval \( R+L \) (Silver, Pyke, Peterson, 1998). As a consequence, the expected shortage function for each item \( i \) is

\[
\text{ES}_i(R + L) = \sigma_{R+L} G(k_i)
\]  

(9)

and the expected shortage function for the whole family at the time of review is

\[
\text{ES}(R + L) = \sum_{i=1}^{n} \sigma_{R+L} G(k_i)
\]  

(10)

Service Point Procedure (Carlson, Miltenburg, 1988)

Input: the service objective, \( p \); the review period, \( R \), the lead time \( L \)

Step 1. Calculate the allowed shortage in the replenishment cycle, for the family of items, \( AL \) with (1).

Step 2. At every review period, if ordering is delayed until next review time, examine the inventory position of each individual item in the family and calculate the expected shortage in a replenishment cycle, \( \text{ES}_i(R \mid L) \), as in (9). Then calculate the family expected shortage for the replenishment cycle \( \text{ES}(R + L) \) as in (10).

Step 3. Compare the total expected shortage for the family per replenishment cycle \( \text{ES}(R \mid L) \) with the total allowed shortage per replenishment cycle \( AL \). If \( \text{ES}(R + L) \leq AL \), ordering could be postponed for at least one review period; if \( \text{ES}(R + L) > AL \), ordering should not be postponed.

Step 4. Allocate the truck capacity to all the items in the family (see 1.5).

Often an item will be reordered when the stock level is above the reorder point, (3). Perhaps other items in the family, reaching a critical level at the time of the review will trigger the replenishment order for the whole family. In this case, the safety stock is higher than the expected safety stock target built in the relationship (3), \( k \sigma \), or \( k \sigma_{R+L} \).

1.5 Allocating the Truckload among the Individual Items in the Family

The order quantities \( \alpha_1, \alpha_2, \ldots, \alpha_n \) allocated to each of the individual items in the family to fill exactly the truck capacity \( V \) satisfy the equation

\[
\sum_{i=1}^{n} \alpha_i = V
\]
The allocation is complex, taking into account the inventory position at the time of review. The allocation issue was raised by inventory managers in a recent session of the course the authors taught (Deaconu, Firica, 2008). An algorithm currently used in practice allocates amounts to each item in the family proportional to the item’s mean demand rate. However, some economic criteria have to be mentioned in deciding the allocation: minimizing the expected total stock remaining when the next reorder is triggered, or maximizing the expected number of sales before the next reorder is triggered, or maximizing the expected elapsed time before the next reorder is triggered. In literature it has been shown that these criteria are equivalent (Mendelsohn et. al. 1980). The last criterion (maximizing the expected elapsed time before the next reorder is triggered) is suggested by the method in Miltenburg (1985), which models the inventory position as a diffusion process and shows that just a part of the allocation should be proportional to the item’s mean demand rates. The following method of allocation of individual items results in: for any item $i$ split the allocation $a_i$ in two parts:

$$a_i = a_{i1} + a_{i2}$$

where, maintaining the notation $y_i$ for the inventory position of item $i$ at the time of the review,

$$a_{i1} = 0.8Q_i - y_i$$

is that amount of item $i$ that needs to be ordered to bring the inventory position to 80% of the expected usage for item $i$, $Q_i$ in the current replenishment cycle, while $a_{i2}$ is allocated based on $D_i$, the annual demand for item $i$:

$$\frac{a_{i1}}{D_i} = \frac{a_{i2}}{D_i} = \cdots = \frac{a_{iN}}{D_i} = \text{const}$$

Note that the ratio

$$\frac{y_i + a_i}{D_i}$$

is constant, whatever $l = 1, 2, ..., n$. Obviously, this allocation strongly relies on the inventory position at the time of replenishment. The 80% level has been chosen to comply with current situation of the family of items and to maximize the expected elapsed time before the next reorder is triggered.

2. Applying the Service Point Model to Creating the Appropriate Tire Stocks

2.1 Short Description of Olympic International Company

Olympic International Company is specialized in tire distribution. Entered on the Romanian market more than 10 years ago, Olympic International focused its activity on the tire import and distribution. The reasons for such a business option are obvious if we take into account several important aspects:

- a dynamic market,
• a strong structure of the force relations between the main actors in this activity field, leaving enough breaches for the small entrepreneurs,

• an affordable financial effort imposed by its entrance and preservation of its position on the market,

• a favorable environment for the development of the small enterprises in Romania (legislation, political message, economic growth, favorable collective mentality),

• a real competence of the entrepreneur in the field (technical knowledge regarding tires, knowledge and sound experience in the internal and external tire sales, managerial skills and experience, a network of business affairs, strong motivation for success).

The analysis of the company’s economic performance in the previous period indicates constantly satisfying results from choosing a strategy of specialization in the distribution of a single tire brand. In the last years, Olympic International has been a creator and active supporter of Continental brand in Bucharest and in the neighbouring area, providing the equipment for the transport fleet of several important companies such as: Transchim, Iveco, Coca-Cola, European Drinks, Vel Pitar, Cristaxi, Romaqua Grup, etc. As one of the factors granting a good relation between a distribution company and its clients and allowing flexibility and rapidity in satisfying the demand is tightly connected to the level and the structure of the tire stock, the management policies of the inventory was analyzed and were substantiated solutions that can be integrated profitably by Olympic International. The coordinated truck transport of the tires of one family from a single point is a current practice at Olympic International, the cost of launching the order being zero and the unitary shipping cost being extremely high. That is why the application of the service point model is suggested.

2.2 The Coordinated Tire Order at Olympic International

In 2008 the sales structure for the 5 families of products distributed by Olympic is presented in table no.1:

<table>
<thead>
<tr>
<th>Tire type</th>
<th>Physical sales (% of the total no. of sold units)</th>
<th>Sales by value (% of the sales value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family 1 Touring cars</td>
<td>60%</td>
<td>22%</td>
</tr>
<tr>
<td>Family 2 Utility vehicles</td>
<td>12%</td>
<td>9%</td>
</tr>
<tr>
<td>Family 3 Trucks</td>
<td>14%</td>
<td>56%</td>
</tr>
<tr>
<td>Family 4 Industrial vehicles</td>
<td>6%</td>
<td>11%</td>
</tr>
<tr>
<td>Family 5 Tubes and flaps</td>
<td>8%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Source: The Data of the Olympic International Company

The family of touring car tires registered the highest physical sales, thus the management analyzes the possibility of coordinated replenishment order for this family made up of three types of car tires (mentioned as Type 1, Type 2, Type 3). The volume allocated to Olympic in a transport is of 400 car tires. The management has the objective to satisfy 90% of the clients’ demand with off-the-shelf products. In 2008 the number of tires sold was of about
5960, which means approximately a monthly demand of 500 pieces. The company reviews the tire stock every two weeks and the lead time is one week. The average demand in the lead time and the standard deviation, high in the case of car tires, can be estimated on the basis of previous recordings. The company has appropriate databases that allow historical evaluations of the demand and other parameters of the model. The data regarding the three types of tires are presented in table no. 2:

**Table no. 2: Initial Data**

<table>
<thead>
<tr>
<th></th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual demand, $D_i$ (pieces)</td>
<td>2880</td>
<td>1920</td>
<td>1200</td>
</tr>
<tr>
<td>Percentage per tire type</td>
<td>48%</td>
<td>32%</td>
<td>20%</td>
</tr>
<tr>
<td>Monthly demand (pieces)</td>
<td>240</td>
<td>160</td>
<td>100</td>
</tr>
<tr>
<td>Expected usage $Q_i$ (pieces)</td>
<td>160</td>
<td>140</td>
<td>100</td>
</tr>
<tr>
<td>$\bar{Q}_{R+L}$, the expected demand during the period $R+L$ (pieces)</td>
<td>90</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>$\sigma_{R+L}$, the standard deviation of demand during the period $R+L$ (pieces)</td>
<td>70</td>
<td>80</td>
<td>15</td>
</tr>
</tbody>
</table>

Source: The Data of the Olympic International Company

We are going to apply the service point procedure. The aggregate acceptable shortage with formula (1) is:

$$AE = \sum_{i=1}^{3} Q_i(1 - p) = (160 + 140 + 100)(1 - 0.90) = 40$$

Suppose that at the review time the inventory position was: 60 tires of type 1, 70 tires of type 2, and 30 tires of type 3. The expected shortage through the replenishment cycle is deduced from table no. 3.

**Table no. 3: The Calculation of the Expected Shortage**

<table>
<thead>
<tr>
<th></th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory position (pieces)</td>
<td>60</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>$\bar{Q}_{R+L}$, the expected demand during the period $R+L$ (pieces)</td>
<td>90</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>$\sigma_{R+L}$, the standard deviation of demand (pieces)</td>
<td>70</td>
<td>80</td>
<td>15</td>
</tr>
<tr>
<td>$k_i$, the safety factor</td>
<td>-0.4286</td>
<td>-0.375</td>
<td>0.6667</td>
</tr>
<tr>
<td>$G(k_i)$ (four d.p.)</td>
<td>0.6493</td>
<td>0.6142</td>
<td>0.1511</td>
</tr>
<tr>
<td>$G^{-1}(k_i)$ (pieces)</td>
<td>45.4534</td>
<td>49.136</td>
<td>2.2665</td>
</tr>
</tbody>
</table>

Source: Authors
By using the formula (10) we get

\[ E[S(R + L)] = 96.856 \]

As

\[ E[S(R + L)] = AL \]

a new order will be launched. We illustrate the way to determine the order quantity for Type 1: \( a_{21} = 1.0 \cdot 100 - 60 = 40 \), and similarly \( a_{21} = 42 \), \( a_{22} = 50 \). However, the sum of orders does not cover the needed 400 tires to be ordered. The other 240 tires left will be allocated as proportional to the annual usage: 116 for Type 1, 76 for Type 2 and 48 for Type 3. Summing up, we get the following allocation, presented in table no. 4.

<table>
<thead>
<tr>
<th></th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{21} )</td>
<td>68</td>
<td>42</td>
<td>50</td>
</tr>
<tr>
<td>( a_{21} )</td>
<td>116</td>
<td>76</td>
<td>48</td>
</tr>
<tr>
<td>( a_{21} )</td>
<td>184</td>
<td>118</td>
<td>98</td>
</tr>
</tbody>
</table>

Source: Authors’ results

The final results of applying the model offer the advantage to substantiate rigorously:

- the decisions based on the established service level (the percentage of the clients immediately served from the existent stock);
- the decision to replenish from producer (quantity, terms, prices);
- the requests of transport means and discounts that the distribution company can negotiate with the producer/transporter;
- the needs of storage surface and the corresponding rental costs.

Conclusions

The scientific stock management is currently a real concern in the tire distribution field, as it offers – especially to the small enterprises – the possibility to act promptly, to be flexible and to satisfy the clients’ demand at a pre-established and satisfactory level. The study carried out within Olympic International Company allowed us to exemplify the utility of the service point model for the coordinated order of multiple similar products transported by truck, from a single distribution point. As the transportation costs are higher, independently from the transported volume, the order coordination leads to cost savings. The model was used exclusively for optimizing the tire stocks of Olympic International Company, but it contains elements that recommend it as useful for other distribution companies whose activity complies with its pattern, in the context of the existence of a real database detailed on products and family of products on previous periods.
References


